Regularity results for anisotropic quasilinear equations

Carlo Alberto Antonini
Università degli Studi di Milano Statale

In this talk I will consider p-Laplace type equations of the form $-\mathrm{div}(a(\nabla u)) = f$ in a possibly anisotropic setting. In particular, I will provide interior Sobolev estimates for the stress field $a(\nabla u)$, as well as weighted L^2 estimates for the Hessian of the solution. This is joint work with Giulio Ciraolo (University of Milan, Italy) and Alberto Farina (Université de Picardie Jules Verne, France).

References

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Classification of radial blow-up at the first critical exponent for the Lin-Ni-Takagi equation in a ball

Denis Bonheure
Université libre de Bruxelles

We investigate the behaviour of radial positive solutions to the Lin-Ni-Takagi problem in the ball $B_R \subset \mathbb{R}^N$:

$$\begin{cases} -\triangle u_p + u_p = |u_p|^{p-2}u_p & \text{in } B_R \\ \partial_\nu u_p = 0 & \text{in } \partial B_R, \end{cases}$$

when $p \to 2^* = \frac{2N}{N-2}$. We obtain a complete classification of finite energy blowing-up solutions to this problem. In particular, we show that if $p \geq 2^*$, then solutions are compact provided that $N \geq 7$. We also give an interpretation of our results in view of the bifurcation analysis of Bonheure, Grumiau and Troestler in Nonlinear Anal. 147 (2016).

Joint work with Jean-Baptiste Castéras (Lisbon), Bruno Premoselli (ULB)

Symmetry for positive critical points of Caffarelli-Kohn-Nirenberg inequalities

Rosario Corso
Università degli Studi di Palermo

We consider positive critical points of Caffarelli-Kohn-Nirenberg inequalities and show a Liouville type result (obtained in [1]) which allows us to give a complete classification of the solutions in a certain range of parameters, providing a symmetry result for positive solutions. The governing operator is a weighted $p-$ Laplace operator, which we consider for a general $1 < p < d$, where d is the dimension of the space. For $p = 2$, the symmetry breaking region for extremals of Caffarelli-Kohn-Nirenberg inequalities was completely characterized by Dolbeault-Esteban-Loss [2]. This is joint work with Giulio Ciraolo (Università degli Studi di Milano Statale, Italy).

References

[1] G. Ciraolo, R. Corso. *Symmetry for positive critical points of Caffarelli-Kohn-Nirenberg inequalities.* Nonlinear Anal. 216 (2022), 112683.

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Clustering Solutions for a Boundary Yamabe Problem

Sergio Cruz Blázquez
Università della Campania "Luigi Vanvitelli"

In this talk we consider a boundary case of the classical Kazdan-Warner problem in dimension greater or equal than four, corresponding to the prescription of scalar and boundary mean curvatures under conformal transformations of the metric. We deal in particular with the case of negative scalar curvature and boundary mean curvature of arbitrary sign, which has been less investigated.

In [1] existence results are obtained using Variational Methods on manifolds of dimension three and nonnegative Escobar class, which are crucially supported by the fact that the blow-up is isolated and simple in that case, but there is no information available for higher dimensions.

We construct *clustering* solutions around boundary critical points of the prescribed curvatures on manifolds with positive Escobar class, showing that the blow-up is not isolated.

This is joint work with Angela Pistoia (University of Rome "La Sapienza", Italy) and Giusi Vaira (University of Bari, Italy).

References

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Multiple critical points for functionals of the calculus of variations without upper growth condition on the principal part

Marco Degiovanni
Università Cattolica del Sacro Cuore

The talk is devoted to variational methods applied to functionals of the calculus of variations of the form

$$f(u) = \int_\Omega L(x, u, \nabla u)\, dx\,.$$

Under suitable *one-sided* conditions on L, it is well known that the functional f admits a minimum point u over $W_0^{1,1}(\Omega)$ (see e.g. [2]).

When there exist more minima, it is natural to raise the question of the existence of mountain pass points and of related solutions of the associated Euler-Lagrange equation.

The fact is well known if L is subjected to suitable *two-sided* conditions (see [4, 5], which in turn develop the celebrated result of [1]).

Following the approach of [3], we aim to obtain a similar result when the functional f has the form

$$f(u) = \int_\Omega \Psi(x, \nabla u)\, dx - \int_\Omega G(x, u)\, dx$$

and the principal part Ψ is subjected only two *one-sided* conditions. In particular, no estimate from above is assumed for Ψ.

For this purpose, nonsmooth variational methods are applied.

This is joint work with Marco Marzocchi (Catholic University of the Sacred Heart, Italy).

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A symmetry result for an elliptic system with doubly critical growth

Francesco Esposito
Università degli Studi della Calabria

In this talk we will consider positive solutions to the following semilinear elliptic system

$$\begin{cases} -\Delta u = \gamma_1 \dfrac{u}{|x|^2} + u^{2^*-1} + \nu\alpha u^{\alpha-1} v^{\beta} & \text{in} \quad \mathbb{R}^n \setminus \{0\} \\ -\Delta v = \gamma_2 \dfrac{v}{|x|^2} + v^{2^*-1} + \nu\beta u^{\alpha} v^{\beta-1} & \text{in} \quad \mathbb{R}^n \setminus \{0\}, \end{cases} \tag{$\mathcal{S}^*$}$$

where $n \geq 3$, $\nu > 0$, $\alpha, \beta > 1$, $\alpha + \beta = 2^*$, with $2^* := 2n/(n-2)$, and $\gamma_1, \gamma_2 \in [0, (n-2)^2/4)$. In particular, we will deduce the radial symmetry of solutions via a careful adaptation of the moving plane plane procedure and making use of the Kelvin transform.

This is a joint work with Rafael López-Soriano (University of Granada, Spain) and Berardino Sciunzi (University of Calabria, Italy).

Some rigidity results for minimal graphs over unbounded Euclidean domains

Alberto Farina
Université de Picardie Jules Verne, Amiens

In the spirit of Bernstein's well-known theorems, I will present some new classification results for minimal Cartesian graphs over unbounded Euclidean domains. In particular, I will show that a positive minimal graph over an open affine half-space, and under the homogenous Dirichlet boundary condition, must be an affine function.

Two flavors are better than one? Some facts about doubly nonlocal equations

Marco Gallo
Università degli Studi di Bari Aldo Moro

In the study of the gravitational collapse of boson stars, the so called *frac-*

tional Choquard equation

$$(-\Delta)^s u + \omega u = \left(\frac{1}{|\cdot|^{N-\alpha}} * F(u) \right) F'(u) \quad \text{in } \mathbb{R}^N \qquad (*)$$

arises naturally, with parameters $s = 1/2$, $\omega > 0$, $N = 3$, $\alpha = 2$ and $F(u) = u^2$. Aim of this talk is to investigate some results about existence and qualitative behavior of solutions of $(*)$ in a more general framework, in particular including the case F' nonhomogeneous and not C^1 (for instance, sublinear).

The treated topics are in collaboration with Silvia Cingolani and Kazunaga Tanaka.

On the number of critical points of solutions of elliptic equations

Massimo Grossi
Sapienza Università di Roma

The calculation of the number of critical points of a solution of a PDE is an old and classic problem. Some powerful techniques (Morse theory, topological degree) allow to give estimates on the total number of critical points. However, the exact calculation requires additional ideas. In the famous Gidas-Ni-Nirenberg Theorem (see [1]) the uniqueness of the critical point of the solution to

$$\begin{cases} -\Delta u = f(u) & in\ \Omega \\ u > 0 & in\ \Omega \\ u = 0 & on\ \partial\Omega \end{cases}$$

in convex and symmetric domains was proved. If we drop the symmetry assumption then the problem because much more difficult and in this case the uniqueness of the critical point was proved only for special f (torsion problem, first eigenfunction).

In this lecture we consider contractible domains allowing that the curvature of $\partial\Omega$ could be negative (see [2], [3]).
Finally the case of the second eigenfunction in domains with large eccentricity will be considered (see [4]).

These are joint works with Fabio De Regibus (University of Roma "La Sapienza') and Francesca Gladiali (University of Sassari, Italy).

References

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Is the optimal rectangle a square?

David Krejčiřík
Czech Technical University in Prague

We give a light talk on an optimality of a square in geometry and physics. First, we recollect classical geometric results that the square has the largest area (respectively, the smallest perimeter) among all rectangles of a given perimeter (respectively, area). Second, we recall that the square drum has the lowest fundamental tone among all rectangular drums of a given area or perimeter and reinterpret the result in a quantum-mechanical language of nanostructures. As the main body of the talk, we present our recent attempts to prove the same spectral-geometric properties in relativistic quantum mechanics, where the mathematical model is a matrix-differential (Dirac) operator with complex (infinite-mass) boundary conditions. It is frustrating that such an illusively simple and expected result remains unproved and apparently out of the reach of current mathematical tools.

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On the uniqueness of positive solutions of the Lane-Emden problem

Isabella Ianni
Sapienza Universià di Roma

The question of the uniqueness for the positive solutions of the Lane-Emden problem arose since the famous symmetry result by Gidas, Ni, Nirenberg (1979), which implies uniqueness when the domain is a ball. A conjecture on the uniqueness in any convex domain was then formulated during the eighties, but only partial answers have been given so far. In this talk we will describe recent results obtained in dimension two about the asymptotic behavior of positive solutions, their non-degeneracy and Morse index computation. In particular we will show that the uniqueness conjecture in convex domains is true in the planar case and for p large enough. We also discuss uniqueness results for the fractional Lane-Emden problem. This is from joint works with Franesca De Marchis, Massimo Grossi, Filomena Pacella (University Sapienza of Roma, Italy), Peng Luo and Shusen Yan (Wuhan Normal University, China), Alberto Saldana (UNAM, Mexico), Abdelrazek Dieb (University Ibn Khaldoun of Tiaret, Algeria).

Symmetry and monotonicity for solutions of vectorial p-Stokes system

Rafael López-Soriano
Universidad de Granada

We shall study qualitative properties of a p-Stokes type system, namely

$$-\boldsymbol{\Delta}_p \boldsymbol{u} = -\operatorname{\mathbf{div}}(|D\boldsymbol{u}|^{p-2} D\boldsymbol{u}) = \boldsymbol{f}(x, \boldsymbol{u}) \ \text{ in } \Omega,$$

where $\boldsymbol{\Delta}_p$ is the p-Laplacian vectorial operator. More precisely, under suitable assumptions on the domain Ω and the vector field $\boldsymbol{f}$, it will be deduced that solutions are symmetric and monotone. Our main results are derived from a vector version of the weak and strong comparison principles, which allow us to apply the moving-planes technique for systems.

This is a joint work with Berardino Sciunzi and Luigi Montoro (Università della Calabria, Italy).

Radial solutions for Pucci-Lane-Emden systems in annuli

Liliane Maia
University of Brasília

In this talk we will present some recent results on existence of positive radial solutions in annuli for a class of nonlinear systems driven by Pucci extremal operators and Lane-Emden coupling in the superlinear regime. Our approach is purely nonvariational. It is based on the shooting method, energy functionals, spectral properties, and on a suitable criteria for locating critical points in annular domains through the moving planes method that we also adapt.
We refer to [1,2].

This is joint work with Gabrielle Nornberg (Universidad de Chile, Chile) and Ederson Moreira dos Santos (Universidade de São Paulo, Brazil).

References

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New Gagliardo-Nirenberg inequalities and applications to biharmonic NLS

Rainer Mandel
Karlsruhe Institute of Technology

We present new existence results for nontrivial standing wave solutions of the biharmonic Nonlinear Schrödinger equation of mixed dispersion type

$$i\partial_t\psi - \Delta^2\psi + 2\Delta\psi = |\psi|^{p-2}\psi, \quad \psi(0,x) = \psi_0(x), \quad (t,x) \in \mathbb{R}\times\mathbb{R}^d.$$

To deduce the existence of orbitally stable standing waves with a given mass as in Cazenave-Lions ('82), we prove a new class of Gagliardo-Nirenberg

inequalities involving the Helmholtz operator instead of the Laplacian. Having explained the relevance of such inequalities for our analysis, we comment on their proofs and related questions from Harmonic Analysis. Finally, we discuss a symmetry-breaking phenomenon for small masses that was recently discovered by Lenzmann and Weth. This is joint work with Antonio Fernández (Instituto de Ciencias Matemáticas Madrid, Spain), Louis Jeanjean (Université de Franche-Comté, France) and Mihai Maris (Universitde Toulouse, France).

Ground states and bound states of $\nabla \times \nabla \times u = |u|^4 u$ in $\mathbb{R}^3$

Jarosław Mederski
Polish Academy of Sciences

We look for solutions $u : \mathbb{R}^3 \to \mathbb{R}^3$ of the critical curl-curl problem

$$\nabla \times (\nabla \times u) = |u|^4 u.$$

Ground states are related with the optimizers of a Sobolev-type inequality involving the curl operator in $\mathbb{R}^3$. We show that there is a ground state solutions and infinitely many bound state solutions. Some symmetric properties of the problem and extensions to the p-curl-curl equation in the critical case will be discussed.

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Classification of solutions to $-\Delta u = u^{-\gamma}$ in $\mathbb{R}^N_+$

Luigi Montoro
Università della Calabria

We provide a classification result for positive solutions to $-\Delta u = 1/u^\gamma$ in the half space, under zero Dirichlet boundary condition.
We refer to [1].

This is a joint work with L. Muglia (University of Calabria, Italy) and B. Sciunzi (University of Calabria, Italy).

References

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Uniqueness of maximum point for the p-torsion function of convex domains in the plane

Sunra Mosconi
Università degli Studi di Catania

Given a convex domain Ω in the plane, we will consider the unique solution of the so-called *Torsion problem*, i. e.

$$\begin{cases} -\Delta_p u = 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

Here $p > 1$, $-\Delta_p = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator and no regularity is assumed on $\partial\Omega$ beyond the natural Lipschitz one.
It is well-known from [4] that the *torsion function* u solving the problem is log-concave, i. e. $\log u$ is concave. When $p = 2$ and in any dimension, $\log u$ is also strictly concave, thus it has a unique maximum point, a property inherited by u itself. This is usually proved through the *Constant Rank Principle* of Caffarelli-Friedman [2,3], ensuring that the rank of $D^2 \log u$ is constant on Ω. Due to the singular/degenerate nature of the p-Laplacian,

the Constant Rank Principle (whenever it can be stated) is no longer true when $p \neq 2$, and different techniques are needed.

By analising the *heart* of the convex body Ω (see [1]) and through the Aleksandrov reflection method, we are able to prove that u attains its maximum at a unique point of Ω. As a byproduct, we derive the strict concavity of $\log u$, hence the strict convexity of the super-level sets of u, even if the domain Ω has large flat parts, as in the case of a square.

The arguments are essentially geometric, relying on the existence of shadow-sections of convex bodies which only holds in dimension 2.

If time permits, we will briefly discuss a similar statement for solutions of $-\Delta_p u = f(u)$ with suitable reactions f and an example showing that our technique has little hope to succeed in dimension greater than 2.

This is joint work with William Borrelli (Catholic University of Brescia) and Marco Squassina (Catholic University of Brescia).

References

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Brezis-Oswald results for fractional and mixed operators

Dimitri Mugnai
Università degli Studi della Tuscia

We show some results in the spirit of the classical one by Brezis-Oswald in

the case of the fractional p-Laplacian and of mixed local-nonlocal operators.

Overdetermined problems and constant mean curvature surfaces in unbounded sets

Filomena Pacella
Sapienza Università di Roma

In this talk we discuss the question of characterizing domains Ω contained in a fix unbounded set $\mathcal{C}$ which admit solutions of a partially overdetermined problem. We present some recent results which allow to describe some properties of Ω, in particular when $\mathcal{C}$ is a cylinder. Related results for the question of characterizing constant mean curvature surfaces inside the set $\mathcal{C}$ will be described. Finally we discuss both problems when $\mathcal{C}$ is a cone, in which case fairly complete results have been obtained.

This research is contained in some joint works with Giulio Tralli, Alessandro Iacopetti, Tobias Weth and Danilo Gregorin Afonso.

Shape optimization of nonlinear eigenvalues and torsional rigidity on perforated domains

Gianpaolo Piscitelli
Università degli studi di Napoli Federico II

In this talk we show an optimal upper bound for the first eigenvalue of mixed Robin-Neumann boundary value problems for the p-Laplacian operator in domains with holes. An analogous estimate is obtained for the corresponding torsional rigidity problem [1,2].

This is joint work with Francesco Della Pietra (Università degli studi di Napoli Federico II, Italy), Gloria Paoli (Friedrich-Alexander-Universitt Erlangen-Nornberg, Germany) and Leonardo Trani (Università degli studi di Napoli Federico II, Italy).

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Segregated solutions for some nonlinear Schrödinger systems

Angela Pistoia
Sapienza Università di Roma

In this talk I will focus on the existence and multiplicity of positive non-radial solutions for some systems of Schrödinger equations in a weak fully attractive or repulsive regime in presence of an external radial trapping. This is a joint work with Haixia Chen (Central China Normal University, Wuhan) and Giusi Vaira (Università di Bari).

Quantitative stability for an overdetermined problem with the fractional Laplacian

Luigi Pollastro
Università degli Studi di Milano Statale

In this talk, I will present a result regarding symmetry and quantitative stability for the fractional torsion problem in a bounded domain $\Omega \subset \mathbb{R}^n$ with an overdermined condition on a a surface parallel to the boundary $\partial\Omega$.

More precisely, the result states that if the fractional torsion function has a C^1 level surface which is parallel to the boundary $\partial\Omega$ then the domain is a ball. If instead one assumes that the solution is close to a constant on a parallel surface to the boundary, then one can quantitatively prove that Ω is close to a ball.

Since the seminal paper of J. Serrin, overdetermined problems have been a popular topic in the PDE community. Starting from the original proof of Serrin, I will highlight the main tools needed in order to achieve symmetry

(the method of moving planes and maximum principles) together with their quantitative counterparts; lastly, I will give some details on the adaptation of these techniques to the new result in order to take into account the nonlocal nature of the problem.

This is a joint work with Giulio Ciraolo (University of Milan, Italy), Serena Dipierro, Giorgio Poggesi and Enrico Valdinoci (University of Western Australia).

Classification and non-existence results for critical $p-$Laplace type equations

Alberto Roncoroni
Politecnico di Milano

The starting point of the seminar is the well-known generalized Lane-Emden equation

$$\Delta_p u + |u|^{q-1}u = 0 \quad \text{in } \mathbb{R}^n\,, \tag{1}$$

where Δ_p is the usual $p-$Laplace operator with $1 < p < n$ and $q > 1$. I will discuss several non-existence and classification results for positive solutions of (1) in the subcritical ($q < p^* - 1$) and in the critical case ($q = p^* - 1$). In the critical case, it has been recently shown, exploiting the moving planes method, that positive solutions to the critical $p-$Laplace equation (i.e. (1) with $q = p^* - 1$) and with finite energy, i.e. such that $u \in L^{p^*}(\mathbb{R}^n)$ and $\nabla u \in L^p(\mathbb{R}^n)$, can be completely classified. In this talk, I will present some recent classification results for positive solutions to the critical $p-$Laplace equation with (possibly) infinite energy satisfying suitable conditions at infinity. Moreover, if time permits I will discuss analogue results in the anisotropic, conical and Riemannian settings.

This is based on a recent joint work with G. Catino and D. Monticelli.

The unbearable weight of massive functions

Jacopo Schino
North Carolina State University

Schrödinger-type equations model a lot of natural phenomena and their solutions have interesting and important properties: one of them is the conserva-

tion of mass, which gives rise to the search for *normalized solutions*. In this talk, I will explain a possible approach in the so-called *strongly sublinear case* (also referred to as the *infinite-mass case*), i.e., when the ratio between the right-hand side and its argument diverges negatively at zero, which makes the usual approach impossible because the energy functional is not well defined. In the proposed approach, a family of approximating problems is considered so that the energy functional is well defined and a corresponding family of solutions is obtained, which eventually converge to a solution to the original problem.

This is joint work with Jarosław Mederski (Polish Academy of Sciences, Warsaw).

Overdetermined elliptic problems in the sphere

Pieralberto Sicbaldi
Universidad de Granada

Consider the general Serrin's overdetermined problem in a simply connected subdomain of the sphere. By a very well known result by Kumaresan and Prajapat, if the subdomain is contained in a hemi-sphere then it must be a geodesic ball. The proof of such result generalizes the classical moving plane technique to the sphere. In this talk we will try to understand what happens in the general case, when the subdomain is not contained in any hemi-sphere.

This is joint work with David Ruiz and Jing Wu (University of Granada, Spain)

New linking theorems with applications to critical growth local and nonlocal elliptic problems with jumping nonlinearities

Caterina Sportelli
Università degli Studi di Bari Aldo Moro, Italy

In this talk we will present some new existence results for critical growth elliptic problems with jumping nonlinearities such as

$$\begin{cases} -\Delta u = bu^+ - au^- + |u|^{2^*-2}\, u & \text{in } \Omega \\ \qquad u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $N \geq 4$, $2^* = 2N/(N-2)$ is the critical Sobolev exponent, $a, b > 0$ and $u^{\pm} = \max\{\pm u, 0\}$.
The provided results are obtained via new abstract linking theorems which are of independent interest and can be used to obtain nontrivial solutions of other types of problems with jumping nonlinearities as well.
Thereby, we will both give an existence result for a particular critical growth problem when N = 2, and we will briefly discuss the nonlocal anologue of problem (2).

These results are part of a joint work with Kanishka Perera (Florida Institute of Technology, USA).
Also, the results related to the nonlocal anaolgue of problem (2) are part of an ongoing work with Kanishka Perera (Florida Institute of Technology, USA), Giovanni Molica Bisci and Raffaella Servadei (Università degli Studi di Urbino Carlo Bo, Italy).

On concavity of solutions to nonlinear PDEs: some new results

Marco Squassina
Università Cattolica del Sacro Cuore

We obtain some new concavity results, up to a transformation, for the positive solutions to some classes of nonlinear problems driven by the $p-$ Laplace operator.

Non-variational weakly coupled elliptic systems

Andrzej Szulkin
Stockholms Universitet

The elliptic system of 2 equations

$$-\Delta u_i + \kappa_i u_i = \mu_i u_i^3 + 2\lambda u_i u_j^2, \quad i, j = 1, 2, \ i \neq j,$$

where Ω is a domain in $\mathbb{R}^N$, has been extensively studied in dimensions $N \leq 4$. This system appears e.g. in 2-species physical problems (Bose-Einstein condensates with 2 hyperfine states) and in population dynamics (Lotka-Volterra equations). In this talk we will be concerned with a system

of ℓ equations which in general is *non-variational*. If $\ell = 2$, the system we consider is

$$\begin{cases} -\Delta u_1 = \mu_1 u_1^p + \lambda_{12} u_1^{\alpha_{12}} u_2^{\beta_{12}}, \\ -\Delta u_2 = \mu_2 u_2^p + \lambda_{21} u_2^{\alpha_{21}} u_1^{\beta_{21}}, \\ u_1, u_2 \geq 0 \text{ in } \Omega,\ u_1, u_2 \in H_0^1(\Omega) \setminus \{0\}. \end{cases}$$

Here Ω is a smooth bounded domain in $\mathbb{R}^N$ $(N \geq 2)$, $1 < p < (N+2)/(N-2)_+$, $\mu_i > 0$, $\lambda_{ij} < 0$, $\alpha_{ij}, \beta_{ij} > 0$ and $\alpha_{ij} + \beta_{ij} < p$. We show, using a combination of variational and topological arguments, that this system possesses at least one solution. If time permits, we also show that under suitable conditions there exist synchronized solutions for $\lambda_{1,2}, \lambda_{2,1} \to -\infty$.

This is joint work with Mónica Clapp (UNAM, Mexico City, Mexico)

Multiple solutions for the nonlinear Choquard equation with even or odd nonlinearities

Kazunaga Tanaka
Waseda University, Tokyo

We prove exsitence of infinitely many radially symmetric solutions $u \in H_r^1(\mathbb{R}^N)$ for the nonlinear Choquard equation:

$$-\Delta u + \mu u = (I_\alpha * F(u))F'(u) \quad \text{in } \mathbb{R}^N,$$

where $N \geq 3$, $\alpha \in (0, N)$, $I_\alpha(x) = \frac{\Gamma(\frac{N-2}{2})}{\Gamma(\frac{\alpha}{2})\pi^{N/2}2^\alpha} \frac{1}{|x|^{N-\alpha}}$, $x \in \mathbb{R}^N \setminus \{0\}$ is the Riesz potential, and F is assumed odd or even and in a wide class of subcritical nonlinearities in a sense of Beresticky and Lions (1983), Moroz and Van Schaftingern (2015). We implements a new construction of multidimensional odd path, where some estimates for the Riesz potential play an essential role. This is a joint work with Silvia Cingolani and Marco Gallo.

On a Donaldson functional for CMC-immersions of surfaces into Hyperbolic 3-manifolds

Gabriella Tarantello
Università degli Studi di Roma Tor Vergata

We discuss a parametrization for the moduli space of Constant Mean Curvature (CMC) immersions of a closed surface (orientable and of genus larger

than 1) into hyperbolic 3- manifolds by elements of the tangent bundle of the Teichmueller space. Such tangent bundle is identified by pairs formed by a conformal structure on the surface and a Dolbeault cohomology class of (0,1)-forms valued in the corresponding holomorphic tangent bundle. Thus, for any such pair, we establish the unique solvability of the Gauss -Codazzi equations, expressing the pullback metric and the second fundamental form of the immersion in terms of the given element of the tangent bundle. The Gauss-Codazzi equations can be viewed as the Hitchin's selfduality equations for a suitable nilpotent SL(2;C)-Higgs bundle, but also as the Euler-Lagrange equation of a suitable functional, introduced by Gonsalves-Uhlenbeck in 2007, and therefore referred as the Donaldson Functional. Thus we show that the given Donaldson functional admits a unique critical point corresponding to its global minimum. Actually, such uniqueness result extends to a more general version of the Donaldson functional and it permits to recover known results (also about minimal Lagrangian immersions) previously obtained via a Higgs-bundle approach. However, with the Donaldson functional in hand, we are able to describe the asymptotic behavior of its minimum when the co-homology classes vary according to some geometrical pursuit . For example for CMC immersions we shall let the constant of the mean curvature approach its limiting value and describe when the immersed CMC surfaces develop "branched" singularities. Similarly, we will analyze the behavior of minima along a whole ray of cohomology classes. Such an investigation is based on the detailed blow-up analysis developed over the years in the context of Liouville type equations. Here however, we encounter new difficulties as blow-up can occur at a point of "collapsing" zeros of the holomorphic quadratic differentials identified by the given cohomolgy classes.

Comparison results for solutions to elliptic problems with mixed boundary conditions

Cristina Trombetti
Università degli studi di Napoli Federico II

Sharp estimates for solutions to Elliptic Problems with Dirichlet and Neumann boundary conditions are well established results since decades. So far not many results have been obtained in the case of Robin (or mixed) boundary conditions, and in this talk, we shall investigate some open questions related to Talenti-type estimates and Faber - Krahn inequality.